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The Rainbow Connection Number of Snail Graphs

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Abstract

Several previous studies have investigated the rainbow connection number. This research aims to determine the rainbow connection number of the snail graph. The snail graph, denoted by Sln , is constructed from a triangular book subgraph BTn whose common spine is amalgamated with the cycle C4, then two pendant vertices are added and attached to one of the end-vertices of the amalgamated spine. The rainbow connection number of a connected graph G, written rc(G), is defined as the minimum number of colors required so that G becomes rainbow connected. This study is a qualitative descriptive research. The method used is literature study, and the data analysis technique is non-statistical data analysis. From the results, it is found that the rainbow connection number of the snail graph is:4 when n=1;4 when n=2;5 when n=3;5 when n=4; and6 for all n≥5.

Abstrak

Beberapa kajian terdahulu tentang bilangan rainbow connection telah dilakukan. Penelitian ini bertujuan untuk menentukan bilangan rainbow connection pada graf siput. Graf siput yang dinotasikan dengan Sln merupakan graf hasil konstruksi dari subgraf buku segitiga BTn yang busur bersamanya diamalgamasi sisi dengan C4, kemudian ditambahkan dua pendant yang dihubungkan di salah satu titik ujung dari busur yang diamalgamasi. Bilangan rainbow connection dari graf terhubung G, ditulis rc(G), didefinisikan sebagai banyaknya warna minimal yang diperlukan untuk membuat graf G bersifat rainbow connected. Jenis penelitian ini adalah penelitian deskriptif kualitatif, teknik yang digunakan pada penelitian ini yaitu studi pustaka dan teknik analisis data yang digunakan yaitu teknik analisis data non statistik. Dari hasil penelitian diperoleh bahwa bilangan rainbow connection pada Graf Siput yaitu bernilai 4 jika n=1; bernilai 4 jika n=2, bernilai 5 jika n=3; bernilai 5 jika n=4,dan bernilai 6 untuk n≥5.

INTRODUCTION

Graph theory is one of the important branches of discrete mathematics with numerous applications, such as in computer science, network security, scheduling, and cryptography. One of the interesting topics in graph theory is the rainbow connection, which was first introduced by (Chartrand et al., 2008).. The rainbow connection number of a connected graph G, denoted by rc(G), is the minimum number of edge colors required so that every pair of distinct vertices in G is connected by a rainbow path (a path in which all edges have distinct colors).

Let G be a nontrivial connected graph, and let c be an edge-coloring function $c: E(G) \rightarrow \{1,2, \dots, k\}$, where k is a positive integer, allowing two adjacent edges in G to have the same color. A path $u - v$, P, in G is called a rainbow path if no two edges in P share the same color. A graph G is said to be rainbow connected if every pair of distinct vertices in G can be connected by a rainbow path. An edge-coloring that makes G rainbow connected is referred to as a rainbow coloring. In other words, if G is rainbow connected, then G is necessarily connected. Similarly, every connected graph admits a trivial edge-coloring, which implies that a rainbow connected graph has a distinctive edge-coloring. The rainbow connection number of a connected graph G, denoted by $rc(G)$, is defined as the minimum number of colors required to make G rainbow connected (Medika, 2013).

The concept of Rainbow Connection, adapted for the protection of confidential information distribution between the government and its agents, illustrates how to establish secure information paths between two agents without any repeated codes. In this context, neither the government nor the agents can cross-verify information related to national security; consequently, every piece of information delivered to an agent must be encrypted. To achieve this goal, a set of distinct codes must be assigned for each information path between two agents. The required number of codes should ensure that no path uses the same code, thereby maintaining secure communication. This principle leads to the notion of the Rainbow Connection number, which represents the minimum number of codes needed to guarantee at least one secure path between every pair of agents involved. Thus, Rainbow Connection can be applied as a conceptual framework for organizing the transmission of confidential information between the government and agents without the risk of exposure to unauthorized parties (Anggalia, 2017).

Research related to the rainbow connection number has been widely conducted. For instance (Syafrizal Sy et al., 2013) investigated the rainbow connection of sun graphs and fan graphs. (Medika, 2013) studied rainbow connection in several graphs, (Shulhany & Salman, 2015), examined the rainbow connection number of diamond graphs, while (Sari, 2017), analyzed the rainbow connection number of line graphs of windmill graphs. (Kumala, 2019) focused on the rainbow connection number of flower graphs (Wm, Kn) and lemon graphs (Len), (Noor et al., 2021) explored the rainbow connection number of snowflake graphs (Snm), and (Maretha et al., 2021) studied the rainbow connection number of H-graphs. (Gusliana, 2023), worked on the strong rainbow connection number of octopus, planter, and Ferris wheel graphs. (Makhfudloh et al., 2023), considered rainbow connection in snail graphs, coconut shoot graphs, and lotus graphs. Recent developments include rainbow connection in bioriented graphs (Wang, 2024), proper strong rainbow connection (Doan & Schiermeyer, 2024), and rainbow connection in book graphs(Medika et al., 2024). A comprehensive survey on rainbow connection and its variants has also been presented in a recent study (Zhao et al., 2024).

Based on the above discussion, the researcher is interested in studying the rainbow connection number of the snail graph (Sln). According to (Akbar & Sugeng, 2021), a snail graph is constructed from a book graph amalgamated with a cycle $C4$, followed by the addition of two pendant vertices on one of the amalgamated vertices. Several previous studies have examined the snail graph from different perspectives, such as graceful labeling (Akbar & Sugeng, 2021), game chromatic number (Abdurahman et al., 2024), and minimum span value (Komarullah, 2023). However, research on the rainbow connection number of snail graphs with the definition of (Akbar & Sugeng, 2021) remains very limited. Although (Makhfduloh et al., 2023) investigated the rainbow connection of snail graphs, they used a different definition of the snail graph, so the results cannot be directly compared with that of Akbar and Sugeng's definition. Based on this background, the research problem formulated in this study is how to determine the rainbow connection number of snail graphs. Accordingly, the objective of this research is to determine the rainbow connection number of snail graphs.

METHODS

The subject of this research covers the examination of the rainbow connection number and the snail graph. The data sources used in this study consist of both primary and secondary sources. The method applied is a literature study, which involves searching, reading, understanding, and analyzing various works of literature, research results, and studies relevant to the research topic (Sugiyono, 2020).

The procedure undertaken in this study is as follows: (1) Explaining the basic concepts, definitions, and notations related to graph theory, including various types of graphs, with a particular focus on the snail graph. (2) Presenting the general definition of the rainbow connection number. (3) Discussing the rainbow connection number specifically in the snail graph.

The data analysis technique used is non-statistical, in which data are presented, tabulated, and then interpreted or concluded. The stages of data analysis in this study include: (1) Defining the snail graph; (2) Implementing edge-coloring; (3) Identifying rainbow paths, rainbow connectedness, and rainbow coloring; (4) Determining the value of (G); (5) Summarizing the analysis of the value of $rc(G)$; (6) Documenting the results.

RESULT AND DISCUSSION

RESULT

Snail Graph

The snail graph Sln (denoted from snail) is a graph constructed from the triangular book subgraph BTn in which its common spine is amalgamated with the cycle $C4$, Then, two pendant vertices are added and attached to one of the end vertices of the amalgamated spine. Consider the snail graph Sln, where n represents the number of shell patterns of the snail. This graph has n + 6 vertices and $2n$ + 6 edges. The vertex set and edge set of the snail graph Sln are given as follows.

$V(Sln) = \{v_i | 1 \leq i \leq n + 6\}\, and$

$E(Sln) = E(Sln) = \{v_i v_{n+3} | 1 \leq i \leq n\} \cup \{v_i v_{n+4} | 1 \leq i \leq n\} \cup$
$\{v_{n+2}v_{n+4}, v_{n+3}v_{n+4}, v_{n+2}v_{n+5}, v_{n+3}v_{n+5}, v_{n+1}v_{n+2}, v_{n+6}v_{n+2}\}.$

As an illustration, consider Figure 1, which depicts the snail graph along with the labeling of its vertices.

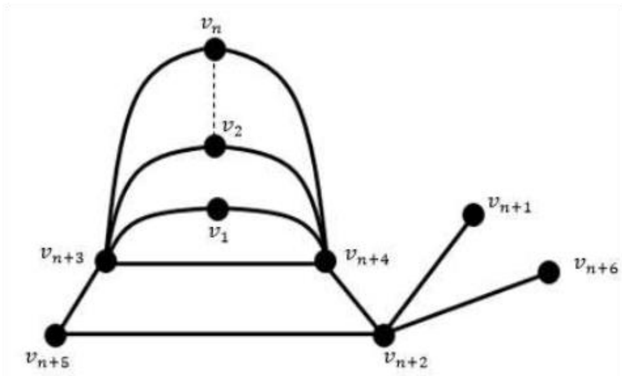

Figure 1. Graf Siput Sln

Theorem 1. For $n \geq 1$, the rainbow connection number of the snail graph Sln is given by

$$rc(Sln) = \begin{cases} 4 + \left\lceil \frac{n-2}{2} \right\rceil, for\ n = 1,2,3,4 \\ 6, for\ n \geq 5 \end{cases}$$

Proof.

The snail graph Sln has $V(Sln) = n + 6$ vertices and $E(Sln) = 2n + 6$ edges. The diameter of this graph is 3, and therefore, in general, it holds that $rc(Sln) \geq 3$

An illustration of Theorem 1 for the snail graph with n=1, is presented in Figure 2.

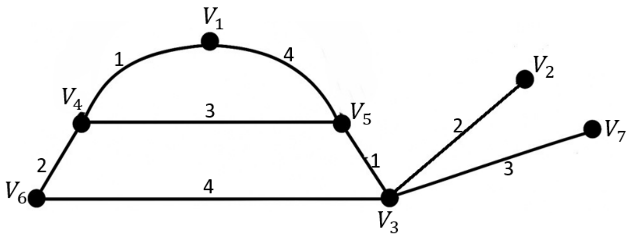

Figure 2. $rc(Sl_1) = 4$

From Table 1, it can be seen that $(Sl_1) = 4$.

Table 1 *The rainbow path* in Sl_1

path	*Rainbow path*	*path*	*Rainbow path*
v_1v_2	412	v_3v_4	42
v_1v_3	41	v_3v_5	1
v_1v_4	1	v_3v_6	4
v_1v_5	4	v_3v_7	3
v_1v_6	12	v_4v_5	3
v_1v_7	413	v_4v_6	2
v_2v_3	2	v_4v_7	243
v_2v_4	213	v_5v_6	32
v_2v_5	21	v_5v_7	13
v_2v_6	24	v_6v_7	43
v_2v_7	23		

Thus, the rainbow connection number of Sl_1 is 4.

The analysis results indicate two distinct cases based on the value of n.
Case 1. For n=1,2 an edge-coloring with four colors is sufficient to guarantee rainbow paths between every pair of vertices. For example, the edges of the cycle $C4$ can be assigned four different colors, while the additional edges in the book part and the pendant vertices are colored in such a way that no path contains repeated colors. Consequently, for n=1,2 we obtain $rc(Sl_n) = 4$.
For n=3,4 the situation changes. The use of only four colors is not sufficient, as there will always exist at least one pair of vertices whose connecting path includes two edges of the same color, thus violating the rainbow condition. However, with five colors, it is possible to construct an edge-coloring where every pair of vertices is connected by a rainbow path. For instance, the edges of the cycle $C4$ can be assigned five distinct colors, and the remaining edges are arranged in a way that avoids color repetition. Therefore, for n=3,4 the rainbow connection number is $rc(Sl_n) = 5$.

Case 2. For $n \geq 5$ neither four nor five colors are sufficient to produce rainbow paths for all pairs of vertices. In such cases, there will always exist a pair of vertices whose connecting path contains repeated colors. The minimum number of colors required is six. If fewer than six colors are used, some paths inevitably contain edges of the same color, preventing the formation of rainbow paths. Thus, for all $n \geq 5$, we conclude that $(Sl_n) = 6$.

DISCUSSION

The findings of this study show that the rainbow connection number of the snail graph increases in small cases (from 4 to 5) but stabilizes at the value of 6 for n≥5. This stability is consistent with the fact that the diameter of the snail graph is 4, which implies that the rainbow connection number cannot be smaller than 4. In the case of Sl_3, the use of five colors is sufficient to ensure that all vertices are connected by a rainbow path. However, for Sl_5, five colors are not sufficient, and thus six colors are required. This indicates the existence of a strong lower bound for larger snail graphs. In addition to differing from the results of (Makhfudloh et al., 2023) due to the different definition of the snail graph, this study also demonstrates consistency with the approach of (Medika et al., 2024a) who applied edge-coloring constructions to determine the rainbow connection of book graphs. Furthermore, the results also open up potential connections to broader concepts, such as the list rainbow connection number (Tang et al., 2025), as well as recent studies on other special classes of graphs (Doan & Schiermeyer, 2024).

CONCLUSION

Based on the results of this study, it can be concluded that the rainbow connection number of the snail graph is 4 when n=1,2 This means that four colors are sufficient to ensure that every pair of vertices is connected by a path whose edges have distinct colors. The value is 5 when n=3,4 indicating that five colors are sufficient to guarantee rainbow paths between all pairs of vertices. For $n \geq 5$ (snail graphs with six shells or more), the rainbow connection number is 6, meaning that six colors are required to ensure that every pair of vertices is connected by a rainbow path.

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